Tip on Force, mass, acceleration units

English system: lbs – Force, ft = Distance → Energy/Work: Ft-lbs.

Metric system kgs = 1000 g = mass

Force (Weight) = (kg)·(9.8... $\frac{m}{s^2}$)

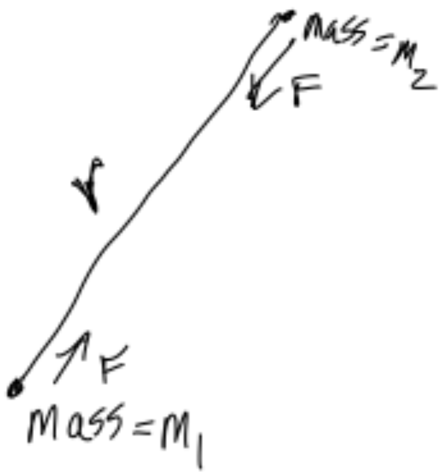
= (mass)(acceleration)

$1 \frac{kg\,m}{s^2} = 1 N$ Newton.

Energy/Work = 1 J Joule

$= 1\,N\cdot m = 1 \frac{kg\,m^2}{s^2}$.

Another type of Work Calculation.

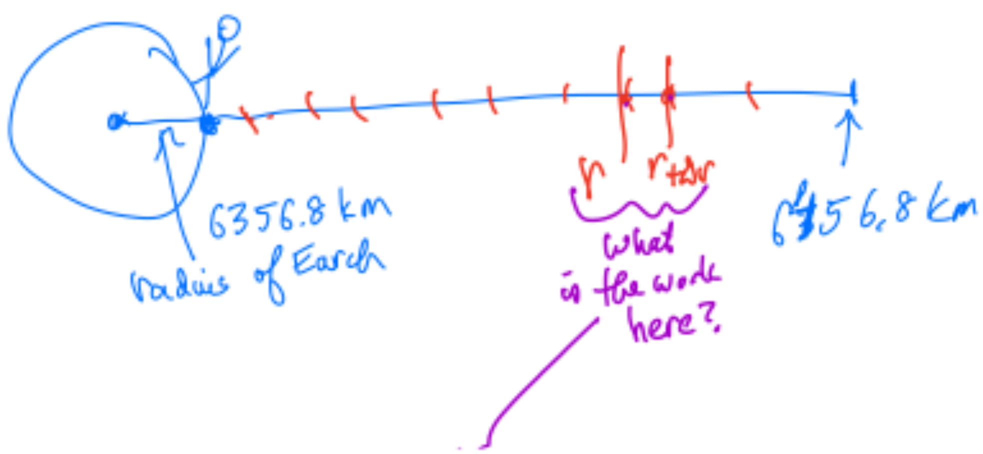


Gravitational Force

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

Example: A giant throws a 3000 kg boulder vertically into space, 100 km into the air. How much Work is required to do that?



$$\text{Force} = \frac{G\, m_1 m_2}{r^2} = \frac{G\,(\overset{5.9736\times10^{24}\,kg}{\text{mass of Earth}})(3000\,kg)}{r^2}$$

$$\text{Distance} = \Delta r$$

Total Work: $W = \int (\text{Force})(\text{distance})$

$$= \int_{6356.8\,km}^{6456.8\,km} \frac{G\, m_1 m_2}{r^2}\, dr$$

$$= G m_1 m_2 \int \frac{1}{r^2}\, dr$$

$$= G m_1 m_2 \left(\frac{-1}{r}\right) \Bigg|_{6356.8\,km}^{6456.8\,km}$$

$$= \left(6.67\times10^{-11}\,\frac{N\,m^2}{kg^2}\right)\left(5.9736\times10^{24}\,kg\right)(3000\,kg)\left(\frac{-1}{6456.8\,km} + \frac{1}{6356.8}\right)$$

$$= (6.67\times10^{-11})(5.9736\times10^{24})(3000)\left(\frac{-1}{\underset{\text{meters!}}{6456800}} + \frac{1}{6356800}\right)$$

$$= 2.91\times10^{9}\ N\cdot m = \boxed{2.91\times10^{9}\ J}$$

Similar Question: Same as above, but the giant throws the boulder into outer space (beyond the pull of the earth.):

$$\text{Work} = \int_{6356.8km}^{\infty} \frac{Gm_1m_2}{r^2}\,dr = \lim_{b\to\infty} Gm_1m_2\left(-\frac{1}{r}\right)\Bigg|_{r=6356.8km}^{b}$$

$$= \lim_{b\to\infty} Gm_1m_2\left(\underbrace{-\frac{1}{b}}_{\to 0}+\frac{1}{6356.8km}\right)$$

$$= \frac{Gm_1m_2}{6356800m} = \frac{\left(6.67\times10^{-11}\frac{Nm^{\cancel{2}}}{\cancel{kg^2}}\right)\left(5.9736\times10^{24}\cancel{kg}\right)\left(3000\cancel{kg}\right)}{6356800\cancel{m}}$$

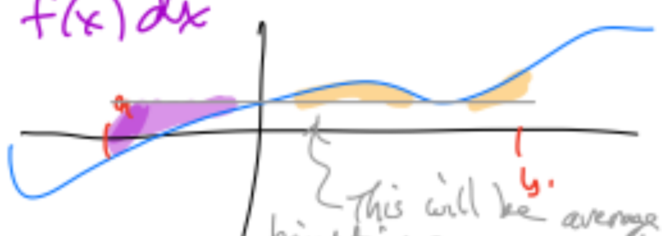
$$= 1.879\times10^{11}\,Nm = \boxed{1.879\times10^{11}\,J}$$

Other applications of integrals:

① Average Value of a function over an interval

$$\begin{pmatrix}\text{Avg of } f(x)\\ \text{for } a\le x\le b\end{pmatrix} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

average height.

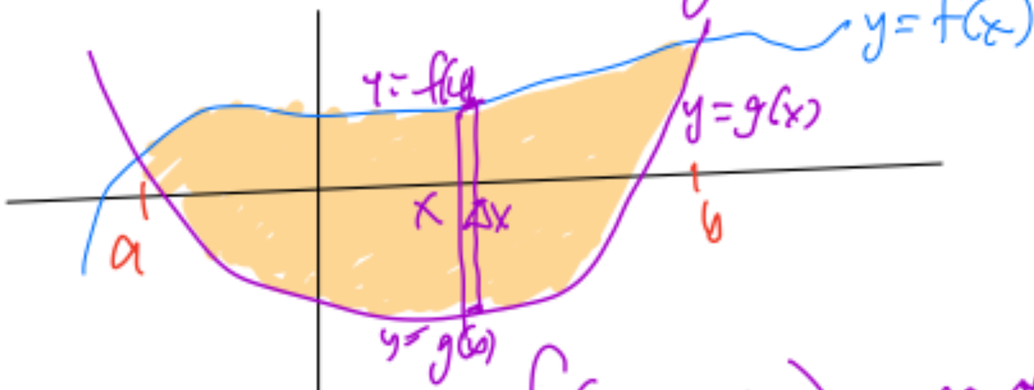


This will be average height if same amount of area is above & below

(2) Average x & y coordinates of a point in a region (x_{cm}, y_{cm}) = center of mass of the region.

x_{cm} = average x-value over region

y_{cm} = average y-value over the region.



$$x_{cm} = \frac{\int (x \text{ of slice}) \cdot \text{area of slice}}{\text{total area}}$$

$$y_{cm} = \frac{\int (y_{cm} \text{ of slice}) \cdot \text{area of slice}}{\text{total area}}$$

$$\Rightarrow X_{cm} = \frac{\int_a^b x \cdot \left[\left(f(x) - g(x)\right)\right] dx}{\int_a^b \left(f(x) - g(x)\right) dx}$$

$$\Rightarrow \quad y_{cm} = \frac{\int_a^b \left(\frac{f(x)+g(x)}{2}\right)\left[\left(f(x)-g(x)\right)\right]dx}{\int_a^b \left(f(x)-g(x)\right)dx}$$